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Cosmology with Interaction between Phantom Dark Energy and Dark Matter and the Coincidence Problem

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Abstract

We study a cosmological model in which phantom dark energy has an interaction with dark matter by introducing a term in the equations of motion of dark energy and dark matter. Such a term is parameterized by a product of a dimensionless coupling function δ, Hubble parameter and the energy density of dark matter, and it manifests an energy flow between the dark energy and dark matter. We discuss two cases, one is that the state parameter ω_e of the dark energy keeps as a constant; the other is that the dimensionless coupling function δ remains as a constant. We investigate the effect of the interaction on the evolution of the universe, the total lifetime of the universe, and the ratio of the period when the universe is in the coincidence state to its total lifetime. It turns out the interaction will produce significant deviation from the case without the interaction.

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1 Introduction

Over the past decade one of most amazing discoveries is the one that our universe is currently in accelerating expansion by observing high red shift supernova Ia [1]. Cross checks confirm this from the cosmic microwave background radiation [2] and large scale structure [3]. To explain this acceleration, one might modify the Einstein's general relativity in the cosmic distance scale or invoke the brane world scenario.

In Einstein's general relativity, in order to give an explanation of the acceleration, one has to introduce a component to the density of the universe with a large negative pressure, which drives the universe to accelerating expand and is dubbed as dark energy in the literature. All astronomical observations indicate that our universe is flat and it consists of approximately 72% dark energy, 21% dark matter, 4.5% baryon matter and 0.5% radiation. A simplest candidate of the dark energy is a tiny positive cosmological constant, which was introduced by Einstein in 1917, two years later after he established general relativity. If it is true, one has to answer the question why the cosmological constant is so small, $\sim 10^{-122}(M_p)^4$, rather than $\sim (M_p)^4$, which is expected from quantum field theory [4]. Here $M_p \sim 10^{19} Gev$ is the Planck mass scale. Although the small cosmological constant is consistent with all observational data so far, recall that a slow roll scalar field can derive the universe to accelerating expand in the inflation model, it is therefore imaginable to use a dynamical field to mimic the behavior of the dark energy. The model of scalar field(s) acting as the dark energy is called quintessence model [5]. Following k-inflation model [6], it is also natural to use a field with noncanonical kinetic term to explain the current acceleration of the universe. Such models are named as k-essence models with some interesting features [7]. Suppose that the dark energy has the equation of state, $p_e = \omega_e \rho_e$, where p_e and ρ_e are pressure and energy density, and ω_e is called state parameter. In order to derive the universe to accelerating expand, one has to have $\omega_e < -1/3$. For the cosmological constant, $\omega_e = -1$; for the quintessence model, $-1 < \omega_e < -1/3$; and for the k-essence model, in general one may has $\omega_e > -1$ or $\omega_e < -1$, but it is physically implausible to cross $\omega_e = -1$ [8].

It is well known that if $\omega_e < -1$, the dark energy will violate all energy conditions [9]. However, such dark energy models [10] are still consistent with observation data ($-1.46 < \omega_e < -0.78$) [11]. The dark energy model with $\omega_e < -1$ is called phantom dark energy model. One remarkable feature of the phantom model is that the universe will end its life with a "big rip" (future singularity) within a finite time. That is, for a phantom dominated universe, its total lifetime is finite. Before the death of the universe, the phantom dark energy will rip apart all bound structures like the Milky Way, solar system, Earth, and ultimately the molecules, atoms, nuclei, and nucleons of which we are composed [12](see also [13]).

Usually people assume that dark energy is coupled to other matter fields only through gravity. Since the first principle is still not available to discuss the nature of dark energy and dark matter, it is therefore conceivable to consider possible interaction between the dark energy and dark matter. Indeed there exist a lot of literature on this subject (see for example [14, 15, 16, 17, 18] and references therein). In this paper, we also consider an interaction model between dark energy and dark matter by phenomenologically introducing an interaction term in the equations of motion, which describes the energy flow between the dark energy and dark matter. We restrict the dark energy is a phantom one. Constraint from supernova type Ia data on such a coupled dark energy model has been investigated very recently [18] (see also [16, 17]). Here we are interested in how such an interaction between the phantom dark energy and dark matter affects the evolution and total lifetime of the universe.

On the other hand, one important aspect of dark energy problem is the so-called coincidence problem. Roughly specking, the question is why the energy densities of dark energy and dark matter are in the same order just now. In other words, we live in a very special epoch when the dark energy and dark matter densities are comparable. Most recently, developing the idea proposed by McInnes [19], Scherrer [20] has attacked this coincidence problem for a phantom dominated universe. Since the total lifetime of the phantom universe is finite, it is therefore possible to calculate the fraction of its total lifetime when the universe in a (coincidence) state for which the dark energy and dark matter densities are roughly comparable. It has been found that the coincidence problem can be significantly ameliorated in such a phantom dominated universe. In this paper we will also study the effect of the interaction between the phantom dark energy and dark matter on the fraction of the period to its total lifetime when the universe is the coincidence state.

The organization of this paper is as follows. In the next section, we first introduce the coupled dark energy model. In Sec. 3 we discuss the case with a constant state parameter ω_e of the phantom dark energy. In Sec. 4 we study the case with a constant coupling function δ introduced in Sec. 2, in this case, the state parameter ω_e will no longer be a constant. The conclusion is contained in Sec. 5.

2 Interacting phantom dark energy with dark matter

Let us consider a universe model which only contains dark matter and dark energy (generalizing to include the baryon matter and radiation is straightforward). A phenomenal model of interaction between the dark matter and dark energy is assumed through an energy exchange between them. Then the equations of motion of dark matter and dark energy in a flat FRW metric with a scale factor a can be written as

$$\dot{\rho}_m + 3H(\rho_m + p_m) = \delta H \rho_m, \tag{2.1}$$

$$\dot{\rho}_e + 3H(\rho_e + p_e) = -\delta H \rho_m, \tag{2.2}$$

where ρ_m and p_m are the energy density and pressure of dark matter, while ρ_e and p_e for dark energy, $H \equiv \dot{a}/a$ is the Hubble parameter, and δ is a dimensionless coupling function. Suppose that the dark matter has $p_m = 0$ and the dark energy has the equation of state $p_e = \omega_e \rho_e$. Note that in general ω_e is a function of time, not a constant. Clearly the total energy density of the universe, $\rho_t = \rho_m + \rho_e$, obeys the usual equation

$$\dot{\rho}_t + 3H(\rho_t + p_t) = 0, \tag{2.3}$$

with the total pressure $p_t = p_e$. The Friedmann equation is

$$H^2 = \frac{8\pi G}{3}\rho_t, \tag{2.4}$$

and the acceleration of scale factor is determined by the equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_t + 3p_t\right), \tag{2.5}$$

where G is the Newton gravitational constant.

In general the coupling function δ may depend on all degrees of freedom of dark matter and dark energy. However, if δ is dependent of the scale factor only, one then can integrate (2.1) and obtain

$$\rho_m = \rho_{m,0} a^{-3} e^{\int \delta d\alpha}, \tag{2.6}$$

where $\alpha = \log a$ and $\rho_{m,0}$ is a constant of integration. Substituting this into (2.2), in order to get the relation between the energy density of the dark energy and scale factor, one has to first be given the relation of the pressure to energy density, namely the state parameter ω_e. Here we follow another approach to study the cosmological model by assuming a relation between the energy densities of dark energy and dark matter as follows:

$$r \equiv \frac{\rho_e}{\rho_m} = \frac{\rho_{e,0}}{\rho_{m,0}}\left(\frac{a}{a_0}\right)^{\xi}, \tag{2.7}$$

where $\rho_{e,0}$, a_0 and ξ are three constants. Set the current value of the scale factor be one, namely $a_0 = 1$, then $\rho_{e,0}$ and $\rho_{m,0}$ have explanation as the current dark energy density and dark matter energy density, respectively.

In this paper we will consider two special cases. One is the case with ω_e being a constant. The other is the case where the coupling function δ is a constant.

3 Cosmology with a constant state parameter of phantom dark energy

In this section we consider the case with a constant ω_e. In this case, one has

$$\rho_e = \frac{Aa^{\xi}}{1+Aa^{\xi}}\rho_t, \quad \rho_m = \frac{1}{1+Aa^{\xi}}\rho_t, \tag{3.1}$$

where the constant $A = \rho_{e,0}/\rho_{m,0} = \Omega_{e,0}/\Omega_{m,0}$, $\Omega_{e,0}$ and $\Omega_{m,0}$ are density parameter values of dark energy and dark matter at present, respectively. The total energy density satisfies

$$\frac{d\rho_t}{da} + \frac{3}{a}\frac{1+(1+\omega_e)Aa^{\xi}}{1+Aa^{\xi}}\rho_t = 0. \tag{3.2}$$

Integrating this yields

$$\rho_t = \rho_{t,0} a^{-3} [1 - \Omega_{e,0}(1 - a^{\xi})]^{-3\omega_e/\xi}, \tag{3.3}$$

where the constant $\rho_{t,0} = \rho_{e,0} + \rho_{m,0}$. Therefore the Friedmann equation can be written down as

$$H^2 = H_0^2 a^{-3} [1 - \Omega_{e,0}(1 - a^{\xi})]^{-3\omega_e/\xi}, \tag{3.4}$$

with H_0 the present Hubble parameter. With (3.1) and (3.3), one can get the coupling function δ from (2.1),

$$\delta = 3 + \frac{\dot{\rho}_m}{H\rho_m} = -\frac{(\xi + 3\omega_e)Aa^{\xi}}{1 + Aa^{\xi}} = -\frac{\xi + 3\omega_e}{\rho_t}\rho_e. \tag{3.5}$$

This can be expressed further as

$$\delta = \frac{\delta_0}{\Omega_{e,0} + (1 - \Omega_{e,0})a^{-\xi}}, \tag{3.6}$$

where $\delta_0 = -\Omega_{e,0}(\xi + 3\omega_e)$. We have $\delta(a \to 1) = \delta_0$ and $\delta(a \to \infty) = \delta_0/\Omega_{e,0}$. Therefore we see that when $\xi > -3\omega_e$, $\delta < 0$, which implies that the energy flow is from the dark matter to dark energy. On the contrary, when $0 < \xi < -3\omega_e$, the energy flow is from the phantom dark energy to dark matter. Further, we can see from (3.5) that there is no coupling between the dark energy and dark matter as $\xi = -3\omega_e$. Of course this is true only for case where ω_e is a constant.

The deceleration parameter q is

$$q \equiv -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{\dot{H}}{H^2} = -1 + \frac{3}{2}\frac{1 - \Omega_{e,0} + (1 + \omega_e)\Omega_{e,0}a^{\xi}}{1 - \Omega_{e,0}(1 - a^{\xi})}. \tag{3.7}$$

Note that $q(a \to \infty) = -1 + 3(1 + \omega_e)/(2\Omega_{e,0})$ and $q(a \to 1) = -1 + 3\omega_e\Omega_{e,0}/2$, they are always negative and $q(a \to 1) < q(a \to \infty)$. In Fig. 1-3 we plot the relation of the deceleration parameter to the red shift defined by $z = 1/a - 1$ for the different ω_e and ξ. In plots we take the density parameter of dark energy as $\Omega_{e,0} = 0.72$. From figures we can see that for a fixed ω_e, a larger ξ leads to a smaller red shift when the universe transits from the deceleration phase to acceleration phase. On the other hand, for a fixed ξ, a larger ω_e corresponds to a smaller red shift for that transition from the deceleration to acceleration phase.

The total lifetime of the universe can be obtained by integrating the Friedmann equation (3.4). It is

$$t_U = H_0^{-1} \int_0^{\infty} da\ a^{1/2} [1 - \Omega_{e,0}(1 - a^{\xi})]^{3\omega_e/2\xi}. \tag{3.8}$$

Here we are interested in the change of the lifetime due to the interaction between the dark energy and dark matter. Note that when $\xi = -3\omega_e$, the interaction disappears. Denote the total lifetime by t_T in this case, one has

$$t_T = H_0^{-1} \int_0^{\infty} da\ a^{1/2} [1 - \Omega_{e,0}(1 - a^{-3\omega_e})]^{-/2}. \tag{3.9}$$

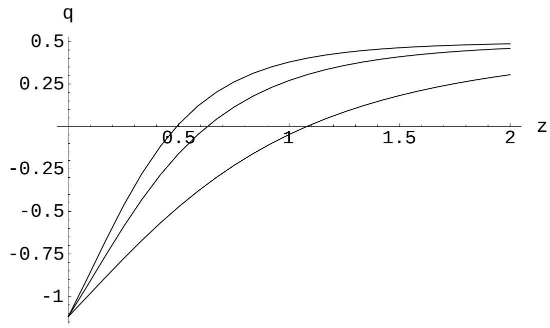

Figure 1: The deceleration parameter q versus the red shift z for the case of $\Omega_{e,0} = 0.72$ and $\omega_e = -1.5$. Three curves from top to bottom correspond to cases $\xi = 5.5$, 4.5 and 3, respectively.

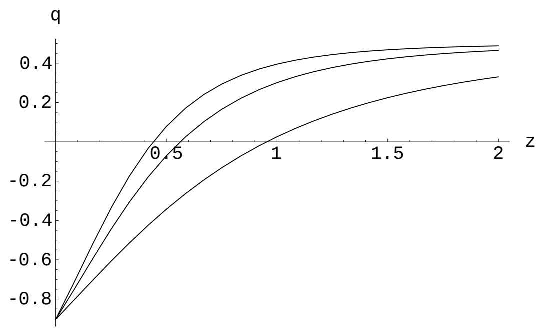

Figure 2: The deceleration parameter q versus the red shift z for the case of $\Omega_{e,0} = 0.72$ and $\omega_e = -1.3$. Three curves from top to bottom correspond to cases $\xi = 5.5$, 4.5 and 3, respectively.

Denote the ratio of the lifetimes t_U to t_T by g:

$$\begin{aligned} g \;\equiv\; \frac{t_U}{t_T} &= \frac{\int_0^\infty da\ a^{1/2}[1-\Omega_{e,0}(1-a^{\xi})]^{3\omega_e/2\xi}}{\int_0^\infty da\ a^{1/2}[1-\Omega_{e,0}(1-a^{-3\omega_e})]^{-/2}} \\ &= \frac{\int_0^\infty \Omega_{e,0}^{-3/2\xi}(1-\Omega_{e,0})^{3(1+\omega_e)/2\xi} r^{3/2\xi-1}(1+r)^{3\omega_e/2\xi}dr}{\int_0^\infty \Omega_{e,0}^{1/2\omega_e}(1-\Omega_{e,0})^{-(1+\omega_e)/2\omega_e} r^{-1/2\omega_e-1}(1+r)^{-1/2}dr}. \end{aligned} \tag{3.10}$$

In Fig. 4 we plot the ratio g for three different state parameters $\omega_e = -1.5$, -1.3 and -1.1. Clearly, for a fixed ω_e, the universe with a larger ξ has a longer lifetime, while for a fixed ξ, a larger ω_e leads to a longer lifetime of the universe. In Fig. 5 the ratio g is plotted versus the parameters ξ and ω_e.

Next we turn to the coincidence problem. Following [20], we calculate the ratio of period when the universe is in the coincidence state to the total lifetime of the universe. That is, we will calculate the ratio

$$f = \frac{t_c}{t_U}, \tag{3.11}$$

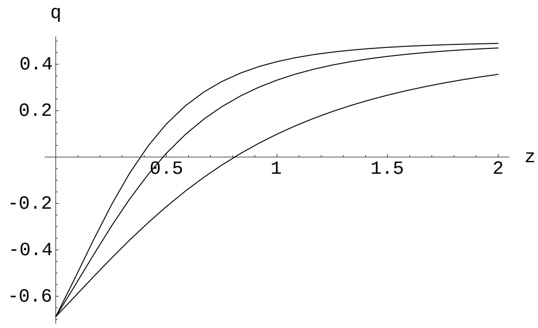

Figure 3: The deceleration parameter q versus the red shift z for the case of $\Omega_{e,0} = 0.72$ and $\omega_e = -1.1$. Three curves from top to bottom correspond to cases $\xi = 5.5$, 4.5 and 3, respectively.

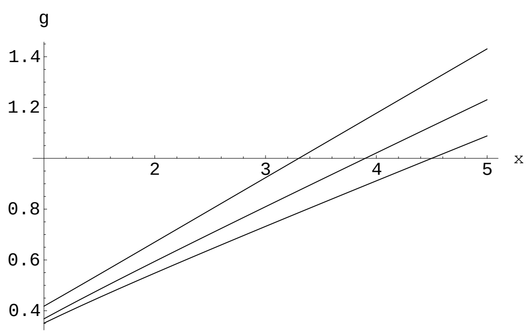

Figure 4: The ratio g of total lifetimes versus the parameter $\xi = x$ for the case of $\Omega_{e,0} = 0.72$. Three curves from bottom to top correspond to the cases $\omega_e = -1.5$, -1.3 and -1.1, respectively.

where the period t_c corresponding to that the universe is in the coincidence state is defined by

$$t_c = H_0^{-1} \int_{a_1}^{a_2} da \ a^{1/2} [1 - \Omega_{e,0}(1 - a^{\xi})]^{3\omega_e/2\xi}. \tag{3.12}$$

During the period of the scale factor from a_1 to a_2, the energy density of dark energy is comparable to that of dark energy. This is not a well-defined problem to determine the scale factors a_1 and a_2. In [20], Scherrer defined a scale of the energy density ratio r_0 so that the dark energy and dark matter densities are regarded as comparable if they differ by less than the ratio r_0 in either direction. He found that the ratio varies from 1/3 to 1/8 as ω_e varies from -1.5 to -1.1 if $r_0 = 10$ in a phantom dark energy model without interaction between the dark energy and dark matter. In this sense indeed the coincidence problem is significantly ameliorated in the phantom model. Now we want to see how the fraction varies when the interaction is present in our model.

The fraction of the total lifetime, when the universe is in the coincidence state, turns out to be

$$f = \frac{\int_{1/r_0}^{r_0} r^{3/2\xi - 1}(1+r)^{3\omega_e/2\xi} dr}{\int_0^{\infty} r^{3/2\xi - 1}(1+r)^{3\omega_e/2\xi} dr}$$

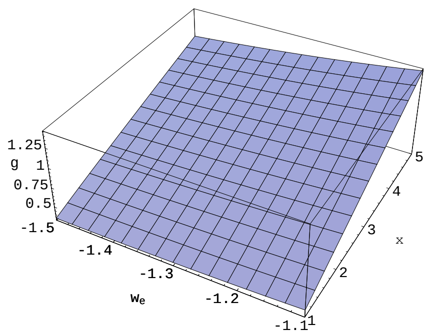

Figure 5: The ratio g of total lifetimes versus the parameters $\xi = x$ and ω_e in the case of $\Omega_{e,0} = 0.72$.

$$= \frac{\frac{2}{3}\left(r_0^{3/2\xi}\ {}_2F_1[\frac{3}{2\xi}, -\frac{3\omega_e}{2\xi}, 1+\frac{3}{2\xi}, -r_0] - r_0^{-3/2\xi}\ {}_2F_1[\frac{3}{2\xi}, -\frac{3\omega_e}{2\xi}, 1+\frac{3}{2\xi}, -\frac{1}{r_0}]\right)}{\frac{\Gamma[3/2\xi]\Gamma[-3(1+\omega_e)/2\xi]}{\Gamma[-3\omega_e/2\xi]}}. \qquad (3.13)$$

Note that this ratio is independent of the current density parameter $\Omega_{e,0}$. In Fig. 6 and 7 we plot the ratio f versus the scale r_0 for different parameters ω_e and ξ. Clearly for fixed ω_e and ξ, a larger r_0 leads to a larger ratio f. For fixed r_0 and ω_e, a smaller ξ gives us a larger ratio. For example, we see from Fig. 6 that when $r_0 = 10$ and $\omega_e = -1.5$, the ratio $f \sim 0.45$ for $\xi = 3$, more large than the case ($f = 1/3$) without the interaction. Note that the middle curve in Fig. 6 corresponds to the case without the interaction ($\xi = -3\omega_e$). In Fig. 8, we plot the ratio f versus the parameters ξ and r_0 for a fixed $\omega_e = -1.3$.

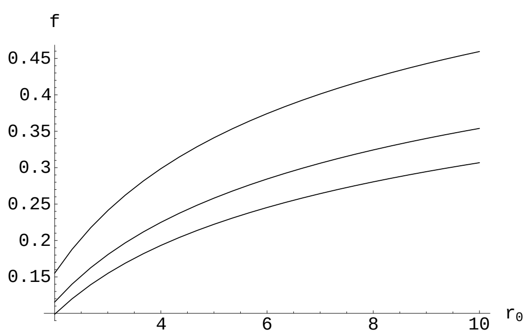

Figure 6: The ratio f versus the parameter r_0 for the case of $\omega_e = -1.5$. Three curves from top to bottom correspond to the cases of $\xi = 3$, 4.5 and 5.5, respectively.

4 Cosmology with a constant coupling parameter

In this section we consider the case with a constant coupling function δ. In this case, we have the energy density of dark matter

$$\rho_m = \rho_{m,0} a^{-3+\delta}. \qquad (4.1)$$

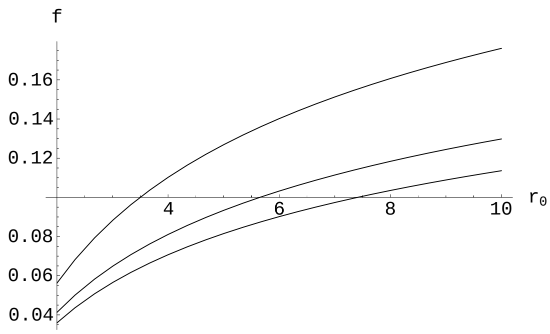

Figure 7: The ratio f versus the parameter r_0 for the case of $\omega_e = -1.1$. Three curves from top to bottom correspond to the cases of $\xi = 2$, 3.3 and 4, respectively.

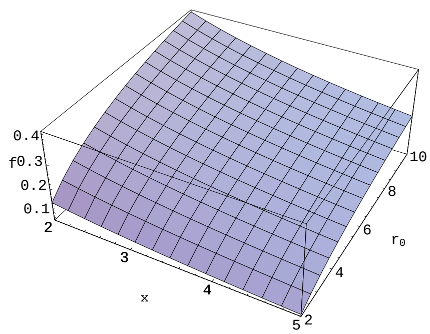

Figure 8: The ratio f versus the parameters r_0 and ξ for the case of $\omega_e = -1.3$. In the figure $x = \xi$.

And then the dark energy density has the relation to the scale factor

$$\rho_e = \rho_{e,0} a^{-3+\delta+\xi}. \tag{4.2}$$

The Friedmann equation turns out to be

$$H^2 = H_0^2(\Omega_{m,0} a^{-3+\delta} + \Omega_{e,0} a^{-3+\delta+\xi}). \tag{4.3}$$

In this case, the state parameter ω_e of the dark energy will depend on time (scale factor). From (2.2), we can obtain

$$\omega_e = -\frac{\delta+\xi}{3} - \frac{\delta}{3}\frac{\Omega_{m,0}}{\Omega_{e,0}} a^{-\xi}. \tag{4.4}$$

When $\delta = 0$, one has $\xi = -3\omega_e$. This situation is just the case without the interaction. From (4.2) and (4.4), one can see that in order the dark energy to be phantom, $\delta + \xi > 3$ so that the dark energy density increases with the scale factor. When $\delta + \xi = 3$, although the dark energy density keeps as a constant, it does not act as a cosmological constant due to the interaction between the dark energy and dark matter. We see from (4.4) that

$$\omega_e(a \to 0) = -sign(\delta)\cdot\infty, \quad \omega_e(a \to 1) = -\frac{\delta+\xi}{3} - \frac{\delta}{3}\frac{\Omega_{m,0}}{\Omega_{e,0}}, \quad \omega_e(a \to \infty) = -\frac{\delta+\xi}{3}. \tag{4.5}$$

The deceleration parameter is found to be

$$q = -1 + \frac{1}{2}\frac{(3-\delta)\Omega_{m,0} + (3-\delta-\xi)\Omega_{e,0}a^{\xi}}{\Omega_{m,0} + \Omega_{e,0}a^{\xi}}, \tag{4.6}$$

which has $q = -1 + (3-\delta-\xi\Omega_{e,0})/2$ when $a = 1$ and $q = (1-\delta-\xi)/2$ when $a \to \infty$. In Fig. 9 and 10 we plot the deceleration parameter versus the red shift for a fixed $\xi = 4$, different δ, and for a fixed $\delta = 0.3$, different ξ, respectively. We see from Fig. 9 that for a given ξ, a larger δ corresponds to a smaller red shift when the universe transits from a deceleration phase to an acceleration phase, while Fig. 10 tells us that for a fixed δ, a larger ξ leads to a smaller red shift.

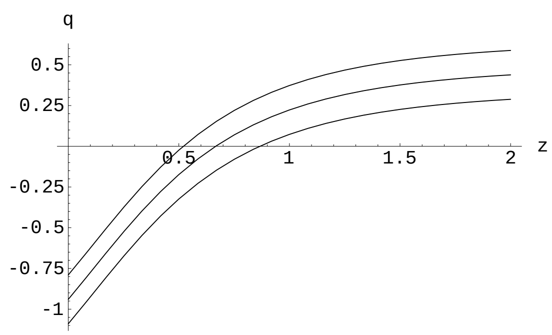

Figure 9: The deceleration parameter q versus the red shift z for the case of $\Omega_{e,0} = 0.72$ and a fixed $\xi = 4$. Three curves from top to bottom correspond to the cases $\delta = 0.3$, 0 and -0.3, respectively.

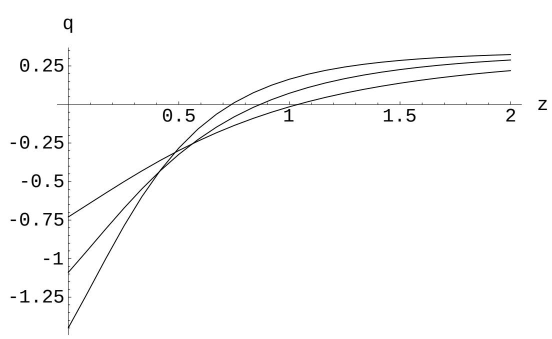

Figure 10: The deceleration parameter q versus the red shift z for the case of $\Omega_{e,0} = 0.72$ and a fixed $\delta = 0.3$. Three curves from bottom to top at the q axis correspond to the cases $\xi = 5$, 4 and 3, respectively.

From (4.3) we can get the total lifetime of the universe

$$t_U = H_0^{-1}\int_0^{\infty} da\ a^{-1}(\Omega_{m,0}a^{-3+\delta} + \Omega_{e,0}a^{-3+\delta+\xi})^{-1/2}. \tag{4.7}$$

We now consider the effect of the interaction on the total lifetime of the universe. Note that the total lifetime of the universe without the interaction is

$$t_T = H_0^{-1} \int_0^{\infty} da\ a^{-1} (\Omega_{m,0} a^{-3} + \Omega_{e,0} a^{-3+\xi})^{-1/2}. \tag{4.8}$$

Denote the ratio t_U/t_T by g, we can express this as

$$g = \frac{\int_0^\infty dr (\frac{1-\Omega_{e,0}}{\Omega_{e,0}})^{(3-\delta)/2\xi} r^{(3-\delta-2\xi)/2\xi} (1+r)^{-1/2}}{\int_0^\infty dr (\frac{1-\Omega_{e,0}}{\Omega_{e,0}})^{3/2\xi} r^{(3-2\xi)/2\xi} (1+r)^{-1/2}}. \tag{4.9}$$

In Fig. 11, the ratio g is plotted versus the parameters ξ and δ for the case $\Omega_{e,0} = 0.72$. We see that the case $\delta > 0$ is quite different from the case of $\delta < 0$. For a fixed $\delta > 0$, a larger ξ leads to a longer lifetime of the universe. On the contrary, for a fixed $\delta < 0$, a smaller ξ gives us a longer lifetime. Further, for a fixed ξ, a smaller δ corresponds to a longer lifetime of the universe.

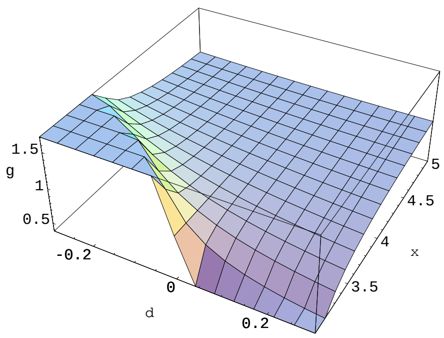

Figure 11: The ratio g of total lifetimes versus the parameters $\xi = x$ and $\delta = d$ for the case of $\Omega_{e,0} = 0.72$.

Finally we consider the ratio of the universe in the coincidence state to the total lifetime. As the case with a constant ω_e considered in the previous section, we calculate the following ratio:

$$f = \frac{\int_{1/r_0}^{r_0} dr\ r^{(3-\delta-2\xi)/2\xi} (1+r)^{-1/2}}{\int_0^\infty dr\ r^{(3-\delta-2\xi)/2\xi} (1+r)^{-1/2}}. \tag{4.10}$$

In Fig. 12 we plot the fraction f versus the scale r_0 for a fixed ξ, but different δ. It shows that a larger δ gives a larger ratio for a fixed r_0. On the other hand, we plot the ratio f in Fig. 13 versus the scale r_0 for a fixed δ, but different ξ, which shows that a larger ξ gives us a larger ratio for a fixed r_0. Fig. 14 shows the relation of the ratio f to the parameters ξ and δ for a fixed scale $r_0 = 5$.

5 Conclusion

In summary we discuss a cosmological model in which phantom dark energy has an interaction with dark matter by phenomenologically introducing a term [see (2.1) and (2.2)] in the equations

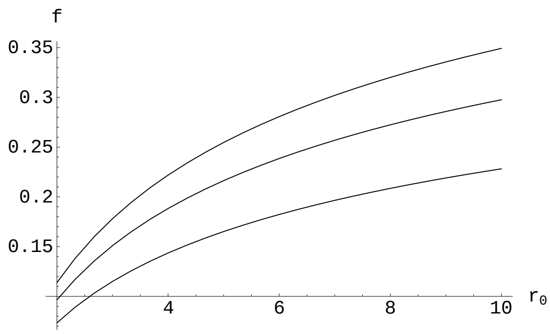

Figure 12: The ratio f versus the parameter r_0 for a fixed $\xi = 4$. Three curves from top to bottom correspond to the cases of $\delta = 0.3$, 0 and -0.3, and corresponding state parameters of dark energy are $\omega_{e,0} = -1.47$, -1.33 and -1.19, respectively.

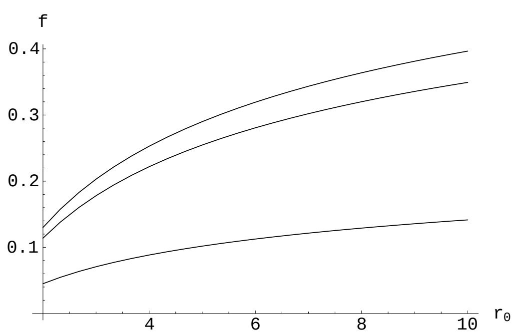

Figure 13: The ratio f versus the parameter r_0 for a fixed $\delta = 0.3$. Three curves from top to bottom correspond to the cases of $\xi = 5$, 4 and 3, and corresponding state parameters of dark energy are $\omega_{e,0} = -1.80$, -1.47 and -1.13, respectively.

of motion of dark energy and dark matter. Such a term is parameterized by a product of a dimensionless coupling function δ, Hubble parameter and the energy density of dark matter, and it manifests an energy flow between the dark energy and dark matter. We discuss two cases, one is that the state parameter $\omega_e = p_e/\rho_e$ of the dark energy keeps as a constant; the other is that the dimensionless coupling function δ remains as a constant. We investigate the effect of the interaction on the evolution of the universe, the total lifetime of the universe, and the ratio of period when the universe in the coincidence state to its total lifetime. We find that the interaction has rich and significant consequence on these issues. For example, the fraction of the period when the universe expands in the coincidence state to its total lifetime can approximately reach 0.45 if we take $\omega_e = -1.5$, $r_0 = 10$ and $\xi = 3$. It turns out that the coincidence problem can indeed be significantly ameliorated in such an interaction phantom dark energy model. Of course, except the constraints on the parameters of the coupled dark energy model from the supernova Ia data [18], further constraints from astronomical observation data, for instance, of cosmic microwave background radiation and large scale structure on this dark energy model

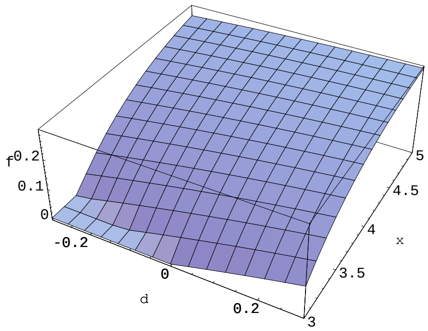

Figure 14: The ratio f versus the parameters $\delta = d$ and $\xi = x$ for a given $r_0 = 5$.

should be carefully studied.

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